AGILE: GIScience Series, 6, 8, 2025. https://doi.org/10.5194/agile-giss-6-8-2025
Proceedings of the 28th AGILE Conference on Geographic Information Science, 10–13 June 2025.
Eds.: Auriol Degbelo, Serena Coetzee, Carsten Keßler, Monika Sester, Sabine Timpf, Lars Bernard
This contribution underwent peer review based on a full paper submission.

Spatial Distribution of Feasible Solutions for Service Facility Location Problems under Uncertainty

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Abstract. After reviewing the existing facility location problems, we discuss the issues arising from uncertainty when applying the optimal solution to actual planning. To address this problem, we propose a method that can increase the degree of flexibility of the facility location planning by finding alternative solutions (facility locations obtained by setting an acceptable tolerance for the value of the objective function). Next, we conduct numerical analysis using hypothetical data and demonstrate the characteristics of facility location problems. Furthermore, the characteristics of the optimal solution and the alternative solutions are examined using numerical analysis in which the proposed method is applied to an actual urban space, and the potential of the proposed method is discussed.

Submission Type. Model, Analysis

BoK Concepts. [AM12-4] Location-allocation modeling and *p*-median problems

Keywords. facility location problem, uncertainty, *p*-median, alternative solution, facility planning

1 Introduction

In planning service facilities, it is important to set the purpose of facility establishment while reflecting the needs of residents, and to plan the facility size, location, service contents, management and operation style, and so on. One of the most important issues to be considered is the location of the facility. For instance, Caprioli and Bottero (2021) pointed out that it is becoming increasingly important to integrate variety kinds of aspects using Multicriteria Decision Analysis (MCDA) for identifying suitable locations for urban infrastructures. For example, the locating problem of new healthcare facilities is very complex because it requires consideration of many aspects, including technical factors, social factors, location factors, and environmental factors. Therefore, they argued that spatial multi-criteria analysis is important to identify the appropriate location of urban facilities.

One of the viewpoints to determine the appropriate location of the facility is the accessibility to the facility. For this reason, there have been many studies using mathematical models on the "facility location problem," which determines the locations of facilities using the distance traveled from locations of users to the facilities as an evaluation index. For example, the facility location problem has been formulated as various optimization problems such as the Weber problem, minimax problem, median problem, center problem, and coverage problem (Jacobsen, 1981; Karatas et al., 2017). As examples of the application of the facility location problem in the field of urban planning, there have been many studies on the location planning of regional facilities for municipalities. Turkoglu and Genevois (2020) pointed out that service facility related location science has attracted great interest in recent years and showed that service facility location problems were classified according to their application areas. Furthermore, Sahin and Sral (2007) reviewed hierarchical facility location models. They classified hierarchical facility problems based on the flow pattern, availability of services at each hierarchical level, and spatial configuration of services, in addition to the objective of locating facilities. They further reported applications of location problems, mixed integer programming models, and solution methods.

In most of the conventional facility location problems proposed so far, the objective function for optimization is composed of travel distances (Kishimoto, 1999). The reason behind this is that it is not only an important factor, but also relatively easy to deal with travel distance quantitatively, although there are differences among Euclidean distance, network distance, and time distance. In urban and regional planning, however, many events are uncertain and difficult to predict at the planning stage. For example, we might discover historical remains in a potential construction site, or a construction site might not be acquired due to lack of consensus with local residents. The factors that are uncertain and difficult to predict cannot be directly evaluated by incorporating them into the optimization model. If the occurrence of such uncertain factors is ignored and a facility planning is drafted based solely on the optimal solution obtained from quantifiable factors, not only will a great deal of time, effort, and mentality be lost, but it will also become more difficult to obtain consensus among decision makers for alternative planning, and the initial plan that was cancelled may become a kind of obstacle for following discussion.

In order to deal with these issues, it is desirable to consider from the outset not only the optimal solution (optimal location), but also alternative feasible solutions that are close to the optimal solution (alternative locations whose evaluated values are close to the optimal solution). Blanco (2019) introduced a new variant of the p-median facility location problem, which assumes that the exact locations of potential facilities are unknown. In this problem, we need to determine the locations of potential facilities to satisfy customer demand that minimizes global costs, while considering the locations of nearby established facilities. He formulated several mixed-integer nonlinear programming methods for objective functions common in location analysis, and developed two mathematical heuristic approaches to solve the problem. His research is based on similar perspectives and has been highly valuable in shaping our new approach in this study. Tirkolaee et al. discussed a robust green location-allocation-inventory problem (LAIP) to design an efficient municipal solid waste (MSW) management system. As the exact quantities of MSW composition in different regions were unknown and uncertain, a robust optimization technique was applied to formulate the problem as a mixed integer linear programming (MILP) model. Also, a real data-based problem was used to validate the proposed model.

Studies focusing on uncertain factors and robustness of facility planning can be roughly divided into two kinds; studies from the side of users of facilities (demand-side) and studies from the side of suppliers of facilities (supply-side). Kubota and Suzuki (2005) is focused on the demand side and discussed facility location-allocation problems when demand is uncertain. The first step in facility planning is to estimate the future population in the target area. Therefore, the errors in population estimation were considered to follow a probability distribution, and the optimal location is analyzed as a single facility allocation using an index that statistically takes into account the errors in the optimal location. Studies focusing on the supply-side include those by Osawa (1996) and Miyagawa et al. (2004). Osawa (1996) compared sequential placement, in which multiple facilities are located sequentially or located simultaneously, in the p-median problem, and demonstrated that there was no significant difference between the two. Miyagawa et al. (2004) examined how the total distance traveled by users changes when facilities are closed sequentially, assuming the decrement of the number of facilities. Sadahiro (2009) applied the method of analyzing the relationship between location distributions to analyze the problem of school consolidation and closure. He proposed a method to extract all solution sets that satisfy the objective function and constraints for school consolidation, and to visually understand the relationship among multiple solution sets.

The studies mentioned above mainly focused on optimization methods, while the following research has specifically examined the impact of uncertainty on optimization problems. Aerts et al. (2003) analyzed how uncertainty in spatial input data propagates through optimization models in Spatial Decision Support Systems (SDSSs) and affects their outcomes. Additionally, they explored the feasibility of routinely incorporating uncertainty propagation methodologies into SDSSs. Furthermore, Wei and Murray (2012) developed an integrated approach to address data uncertainty in spatial optimization. Their study demonstrated that by constructing a novel multi-objective model that explicitly incorporates data uncertainty, it is possible to characterize its impact and evaluate it with statistical reliability. Hildemann et al. (2023) investigated the influence of uncertainty in spatial data—specifically, soil and water conservation measures—within a multi-objective optimization problem that considered both soil runoff rates and labor costs.

Previous research on spatial optimization under uncertainty has primarily focused on quantitatively assessing the degree to which uncertainty affects optimization solutions. In contrast, this study assumes that uncertainty cannot be quantified and instead explores the research question: "Is it possible to propose an alternative solution when the optimal solution cannot be adopted?" Specifically, this study aims to develop a method for identifying candidate locations that serve as viable alternative solutions in cases where uncertainty is too significant to allow for the adoption of the optimal solution. This approach represents the unique contribution of this research. More specifically, in order to deal with the unpredictable uncertainty in the facility location problem, we consider the facility location problem in which redundancy is expected in the objective function for optimization, so that the obtained solution also has redundancy (higher degrees of flexibility) and, as a result, the robustness of the proposed facility planning can be improved.

2 Facility Location Problem Considering Multiple Location Alternatives

2.1 Overview of *p*-median Problem

Consider the p-median problem, one of the representatives of the facility location problem, where t is the number of locations where facility users are located (hereinafter called demand locations) and u is the number of locations where facilities can be located (hereinafter called candidate facility locations). The set $I = \{1, ..., i, ..., t\}$ and the set $J = \{1, ..., j, ..., u\}$ be the sets of indices representing demand locations and candidate facility locations. The p-median problem is the problem of determining the location set of the p facilities, M, and the allocation of users to the facilities so that the total distance traveled by facility users to the facilities (total cost: sum of travel distances weighted by the number of users) is minimized. This problem is formulated as a combinatorial optimization problem, where p locations are selected from u candidate facility locations, as follows. This optimization problem can be solved by minimizing the total cost, Z_M. The set of facility locations, M, that minimizes the value of Z_M represents the optimal solution, which will hereafter be denoted as M^*.

$$
\begin{aligned}
&\min\ Z_M \\
&Z_M = \sum_{i \in I} \sum_{m \in M} P_i C_{ilm} \delta_{ilm} \\
&s.t. \sum_{m \in M} \delta_{ilm} = 1 \ \ for\ \forall i, \qquad \delta_{ilm} \le l_m \ \ for\ \forall i, m, \qquad |M| = p \\
&P_i : \text{demand at location } i \\
&C_{ilm} : \text{Distance from demand } i \text{ to facility at location } l_m \\
&\delta_{ilm} : \begin{cases} 1 : \text{if the closest facility to } i \text{ is located at } l_m \\ 0 : \text{if the closest facility to } i \text{ is not located at } l_m \end{cases} \\
&I = \{1, ..., i, ..., t\} : \text{set of demand locations} \\
&J = \{1, ..., j, ..., u\} : \text{set of candidate facility locations} \\
&M = \{l_m \mid m = 1, ..., p; l_m \in J\} : \text{facility locations}
\end{aligned}
\qquad (1)
$$

In addition to the p-median problem, the p-center and the maximal covering location problems are also problems of determining p locations from u candidate facility locations, and although these three problems have different objective functions, they are the same type of combinatorial optimization problem. The p-center problem is a problem of minimizing the distance from a facility to the furthest demand location, and the maximal covering location problem (MCLP) is to maximize the amount of demand entering a certain area from the facility.

2.2 Methods for Obtaining Multiple Facility Location Alternatives

In order to find multiple alternative locations, it is necessary to simultaneously consider the combination of locations where the facilities are to be located and the value of the objective function (total cost) under the combination of sites. Therefore, we formulate a p-median problem that takes these two pieces of factors into account. The basic idea is to find an exact optimal solution (optimal locations and minimized total cost) in advance, set an "*allowable cost*" by setting a "*tolerance value*" that represents the degree of deviation from the optimal solution, and find a set of combinations of locations that fall within this allowable cost. Specifically, as in the usual facility location problem, all sets consisting of combinations of p locations are considered from the set J of candidate facility locations, and the set M^* (the optimal solution) of the p locations with the lowest total cost is obtained. The total cost of this optimal solution multiplied by the allowable value α ($\geqq 100$) is defined as α % cost (allowable cost), and the set of locations (alternative solution) whose total cost is less than or equal to α % cost no matter which combination of locations is used is obtained as the set M. The set M that is the alternative solution is called the "α % set."

The proposed method is unique in that the allowable cost is a constraint condition, whereas the conventional facility location problem attempts to minimize the total cost by using the total cost as the objective function. In addition, the degree of flexibility of the facility locations (robustness of the facility planning) and the number of candidate locations (the number of locations in the set M or the number of combinations of locations) are used as the objective function for optimization. In other words, the problem is formulated as a problem of finding a larger number of candidate viewpoints under the constraint of satisfying the allowable cost, although somewhat more than the optimal total cost. In the following, we formulate the problem of finding the α % set from following two viewpoints.

2.3 Formulation of *K*-set Problem

The K-set problem is a problem to extract the alternative solutions from a set of Q candidate locations. Specifically, the set of locations for which the total cost is less than or equal to α % no matter how p locations are selected from among Q locations is extracted as the alternative solution. Here, the objective function is the number of locations Q, and the optimal solution is obtained by maximizing Q. The reason for this is that it is desirable to have a large number of locations Q in the K-set when considering multiple location alternatives. The problem is formulated as follows. The constraints state that the number of candidate locations consisting of p locations in the K-set must be less than or equal to α % cost.

$$
\begin{aligned}
&\max\ Q \\
&Z_M \le Z_{M^*} \times \frac{\alpha}{100} \ \ for\ all\ M \\
&s.t. \sum_{m \in M} \delta_{ilm} = 1 \ \ for\ \forall i, \quad \delta_{ilm} \le l_m \ \ for\ \forall i, m, \quad |M| = p \\
&Z_M = \sum_{i \in I} \sum_{m \in M} P_i C_{ilm} \delta_{ilm} \\
&M = \{ l_m \mid m = 1, \dots, p;\ l_m \in K \} : \text{facility locations} \\
&K = \{ l_k \mid k = 1, \dots, Q;\ l_k \in J,\ \ p \le Q \le u \} : \text{set of candidate locations } (K\text{-set})
\end{aligned}
\tag{2}
$$

2.4 Formulation of *R*-set Problem

The R-set problem is different from the K-set problem in that we extract a single K-set as candidate locations, while the R-set as the union of multiple subsets R_m. Specifically, it is formulated as a problem of setting p subsets R_m (a set of T_m locations) and finding a R-set such that the total cost is less than or equal to α % cost no matter how the locations are combined (p in total) by selecting one location from each subset R_m. The objective function here is the number of combinations of the number of locations in each subset. The reason for this is that a large number of combinations of locations is desirable when considering multiple layout alternatives, as it allows for greater flexibility in formulating the facility planning. The problem is formulated as follows.

$$
\begin{aligned}
&\max\ T \\
&T = \prod_{m=1}^{p} T_m \\
&Z_M \le Z_{M^*} \times \frac{\alpha}{100} \ \ for\ all\ M \\
&s.t. \sum_{m \in M} \delta_{ilm} = 1 \ for\ \forall i, \ \ \delta_{ilm} \le l_m \ \ for\ \forall i, m, \ |M| = |R| = p \\
&Z_M = \sum_{i \in I} \sum_{m \in M} P_i C_{ilm} \delta_{ilm} \\
&M = \{ l_m \mid m = 1, \dots, p;\ l_m \in R_m \} : \text{facility locations} \\
&R_m = \{ l_r \mid r = 1, \dots, T_m;\ l_r \in J,\ T_m \le p \} : \text{subset of } R \\
&R = \{ R_m \mid m = 1, \dots, p \} : \text{set of candidate locations } (R\text{-set})
\end{aligned}
\tag{3}
$$

The constraints state that the R-set consists of subsets R_m, and that one location can be arbitrarily extracted from a corresponding subset R_m, so that a total of p locations constitutes the solution set M.

These three solutions to the p-median problem can be summarized as follows: in a conventional p-median problem, the facility locations are uniquely determined as the optimal solution (Fig. 1(a)). One of the alternative solutions can be obtained, by arbitrarily selecting p locations from the K-set (Fig. 1(b)). Similarly, by selecting locations (p in total) from the corresponding subset R_m that constitutes the R-set, another alternative solution can be obtained (Fig. 1(c)).

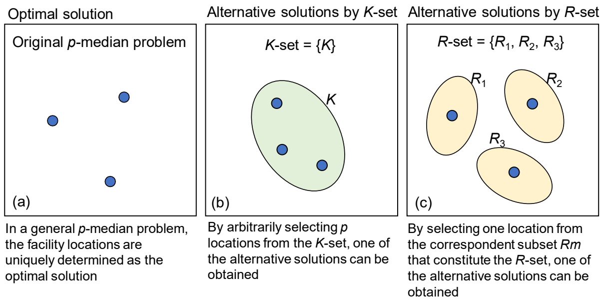

Figure 1. Concept of optimal solution and alternative solutions.

3 Case Studies using Hypothetical Data Set

3.1 Optimal Facility Locations of *p*-median Problem

Figure 2(1) shows a hypothetical data set in which the population (demand) is randomly assigned to a two-dimensional space consisting of 20 x 20 cells (cell size: 500 m). Figure 2(2) shows the results of solving a p-median problem by conventional optimization method with 2 facilities for this hypothetical space. The total cost to the consumer is minimized when the facilities are located at these two locations (optimal solution). However, as mentioned above, it is not always possible to construct facilities at the optimal locations when actual facility planning is assumed.

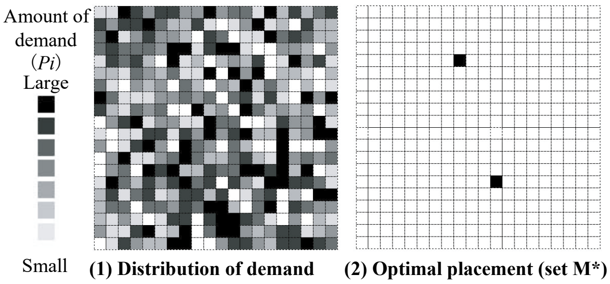

Figure 2. Optimal facility locations of p-median problem.

3.2 Alternative Solutions by *K*-set

Figure 3(1) shows the results of obtaining the K-set with α = 120 in the hypothetical space of Fig. 2(1). It can be seen that the candidate locations, which are the elements of the K-set, are dispersed in the central part. In addition, the K-set does not include the optimal locations obtained by the conventional method. This indicates that even the optimal locations that minimize the total cost deteriorates significantly when combined with other locations that are not members of optimal locations. In other words, the optimal solution obtained by solving the conventional p-median problem shows that when one of the locations is forced to change, the total cost increases significantly and the value of objective function becomes larger. Hence, the robustness of the facility planning based on the optimal solution in this numerical example is not high. Then, is K-set superior? K-set does not include the optimal location, which is undesirable because it means that the optimal locations are abandoned from the beginning. In other words, the K-set does not provide a satisfactory solution for the purpose of optimization.

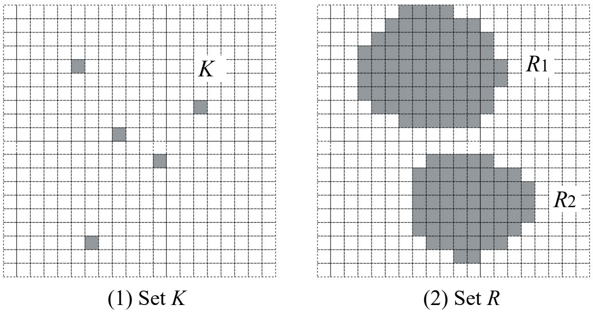

Figure 3. Alternative solutions based on K-set and R-set problems (α = 120).

3.3 Alternative Solutions by *R*-set

Figure 3(2) shows the result of obtaining the R-set in the hypothetical space of Fig. 2(1) with α=120. The total cost of selecting one location from each of the subsets R_1 and R_2 is guaranteed to be less than or equal to 120% of the minimum cost. Although there is a restriction that one location must be selected from subsets R_1 and R_2 respectively, this solution is superior to the K-set in that the number of location combinations is much larger (greater degree of flexibility). This example is also superior in that it is a set that includes the optimal solution (set M^* of optimal locations) obtained by solving the conventional p-median problem. Compared to the K-set, it may contribute to the process of developing a facility planning that is more robust.

Figure 4 shows the α % set when the value of α is varied from 105 to 120 in 5% increments based on the R-set. Figure 5 shows the results of these superimpositions. The spatial characteristics of each subset can also be seen, such as the greater degree of flexibility of R_1 (greater number of locations) compared to subset R_2.

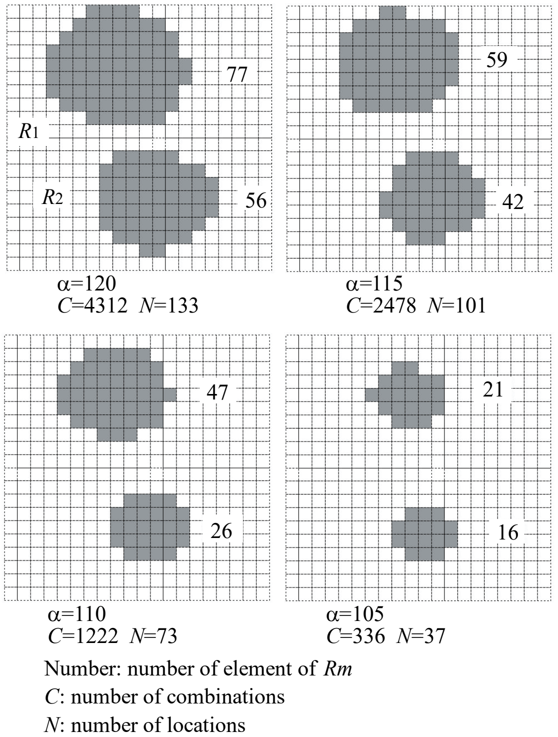

Figure 4. Alternative solutions based on the R-set (α = 105~120).

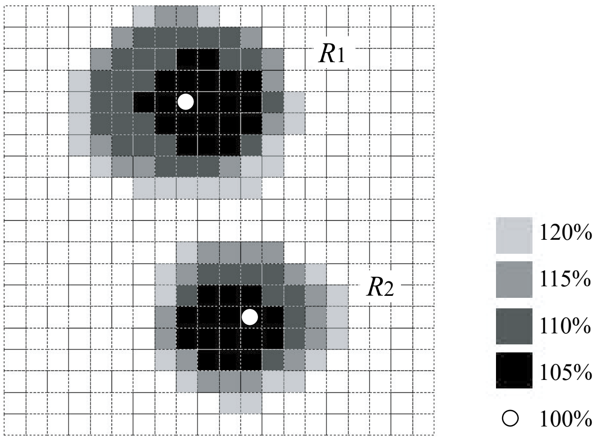

Figure 5. Inclusion relationship of alternative solutions based on the R-set.

4 Case Studies for Tokyo Metropolitan Area

4.1 Characteristics of Facility Location Problems

The following case study is based on the Tokyo metropolitan area. Specifically, we will examine the location of core facilities, such as advanced emergency medical centers, where proximity to residential areas is crucial for accessibility. In this case study, we consider the facility location problem in a network space in order to take into account spaces where there are obstacles that hinder spatial movement (transportation). Specifically, optimal solutions and alternative solutions (α% set based on the R-set) of the p-median, p-center, and maximal covering location problems are obtained, and similarities and differences between three facility location problems are discussed from the viewpoint of not only optimal solutions but also alternative solutions.

The p-median, p-center and maximal covering location problems are the combinatorial optimization problems, each defined by a distinct objective function. The objective function of p-median problem is to minimize the total cost from the demand locations to the facilities, while p-center problem is to minimize the distance of the furthest demand location from the facility. The maximal covering location problem maximizes the amount of demands that falls within a radius of X (in this case, 20 km) centered on the facilities.

The maximal covering location problem is highly similar to the p-median problem, and the optimal solution of the maximal covering location problem can provide good values in the objective function of the p-median problem. For this reason, the p-median problem is often used in general case studies of regional facility allocation. However, for facilities for which a certain service area is important, such as special facilities such as firefighting facilities or emergency hospitals, the maximal covering location problem is often used. On the other hand, the p-center problem is often used to plan the layout of relay facilities in communication networks, etc. The p-center problem is unique in that it is often applied to the layout planning of facilities where the maximum distance between the demand location and the facility is important, without considering the demand volume.

4.2 Target Area and Data

The area to be analyzed is the cities, wards, towns, and villages in the Tokyo metropolitan area (TMA) located within a radius of 70 km around the Imperial Palace. The population of the municipalities in the Tokyo metropolitan area was obtained from the national census (2004), and its spatial distribution is shown in Fig. 6. The discussion here assumes facilities whose demand is proportional to the size of the population. A Delaunay network was constructed based on representative nodes in each municipality, and links that pass through the Tokyo bay were deleted to construct a pseudo traffic network data (Fig. 7). This section discusses the characteristics of the optimal solution and alternative solutions to the facility location problem using the pseudo traffic network that imitates the traffic network in the city.

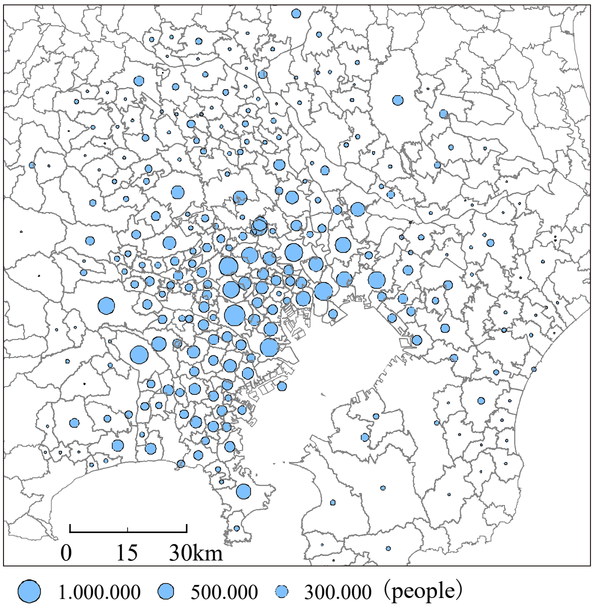

Figure 6. Population distribution in Tokyo metropolitan area.

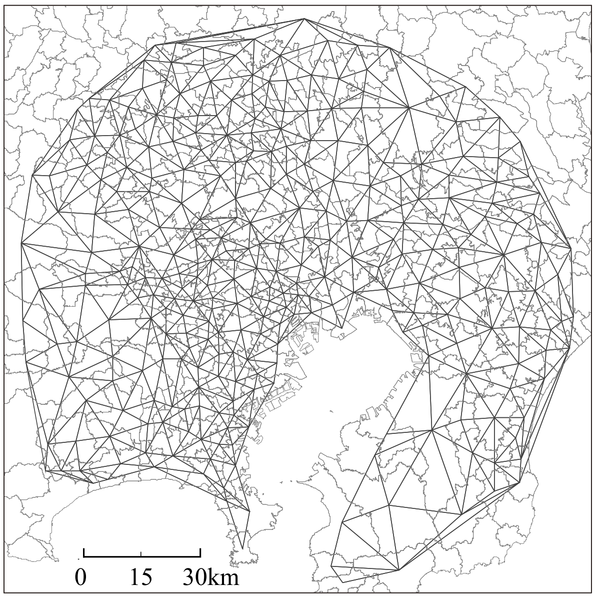

Figure 7. Delaunay triangulation in study area.

4.3 Optimal and Alternative Solutions

Optimal solutions and alternative solutions (α% set based on the R-set) were obtained for the p-median, p-center, and maximal covering location problems when the number of facilities p is 2, 3, or 4. The values of the objective functions under the optimal solutions are shown in Table 1. These values are used to set the tolerance values (allowable cost) when obtaining the alternative solutions. The characteristics of the alternative solutions are discussed below.

Table 1. Objective function value of optimal solution.

Number of facilities	Ave. dist. (km) *p*-median	Max. dist. (km) *p*-center	Coverage (people) MCLP
2	21.9809	70.1285	19,509,334
3	18.6165	58.2897	22,831,241
4	16.4301	46.0929	25,240,295

4.3.1 *p*-median problem

Figure 8(1) shows the distribution of the alternative solutions for the *p*-median problem. It can be seen that the candidate locations of the optimal solutions for p = 2, 3, and 4 are close to each other. The alternative solutions are distributed in almost concentric circles with the optimal solution at the center, indicating that the distribution of the locations (α % set) that become the alternative solutions does not differ greatly when the number of facilities is varied. This result indicates that it is more

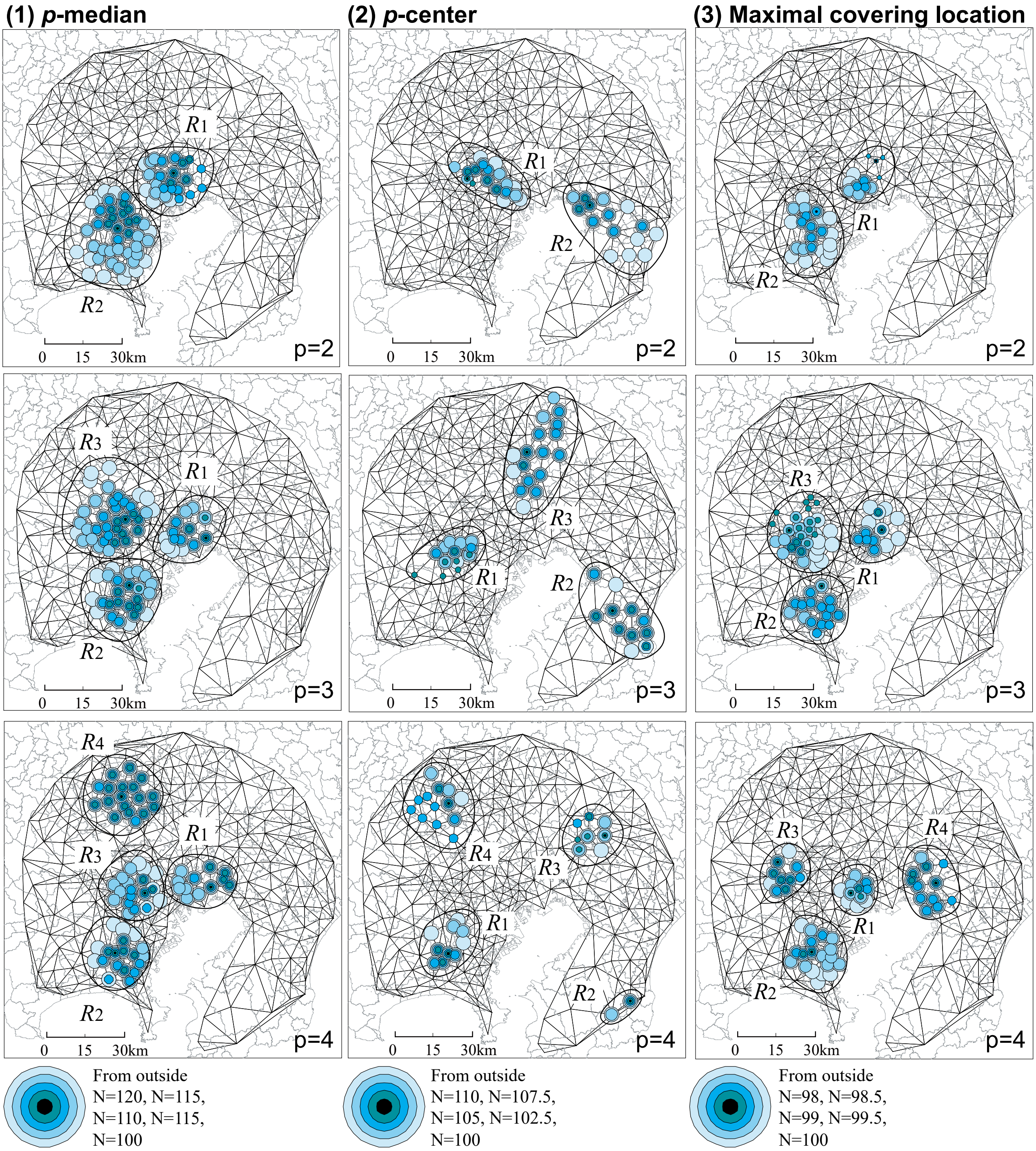

Figure 8. Alternative solutions for facility location problems.

efficient (i.e., the total cost is smaller) to locate facilities concentrated in the urban center because the population is concentrated in the urban center.

A closer look at the subset R_m reveals that alternative solutions are formed at locations where the network is dense. In particular, focusing on R_1, the alternative solutions are not distributed in the east direction, but are clustered in areas where the network is dense. For subsets R_2 and R_3, we can see that they are widely distributed in areas with dense networks and large populations.

4.3.2 *p*-center problem

Figure 8(2) shows the distribution of the alternative solutions of the *p*-center problem. Since the *p*-center problem does not take the quantity of demand into account, it is considered that the results depend on the shape of the space to be analyzed, which is concentric in shape.

In detail, in the results for $p = 2$, one of the optimal solution locations is located slightly west of the center, and the alternative solutions are distributed extending eastward from the optimal solution. In the $p = 4$ result, the subset R_2 is distributed in a localized area. This is related to the sparseness of the network in the vicinity of subset R_2. Specifically, this is because the search for a alternative solution in the spatial alternative of the optimal solution results in a large spatial separation, resulting in a large deterioration of the value of objective function. Compared to the other three subsets R_1, R_3, and R_4, the degree of flexibility of the locations is extremely low. In contrast, R_2 with $p = 3$ has a wider distribution of alternative solutions than that with $p = 4$, even though it is distributed in an area with a sparse network. This indicates that there is higher degree of flexibility in the selection of facility locations when the number of facilities to be located is 3 than is 4.

4.3.3 maximal covering location problem

The solution to the maximal covering location problem is shown in Fig. 8(3). *p*-median and *p*-center problems are problems that seek to minimize the value of the objective function, so the allowed value of the alternative solution is $\alpha \geqq 100$. Note, however, that for the maximal covering location problem, which aims to maximize the value of the objective function (the total demand that can be covered), the allowable value of α is $\alpha \leqq 100$.

An overview of Fig. 8(3) shows that both the optimal solution and the alternative solutions are concentrated and distributed in urban centers, similar to the solution of the *p*-median problem. However, while the alternative solutions of the *p*-median problem are distributed adjacent to each other, the subset R_m of the alternative solutions of the maximal covering location problem is distributed with some spatial interval. This is because the subset R_m is formed so that the area covered by each facility (20 km in this case) does not overlap. In the *p*-median problem, the optimal solution is located at the center of the candidate locations, but the maximal covering location problem is characterized by the fact that in many cases the optimal solution is located at the edge of the candidate locations.

4.4 Facility Location Problem Considering Existing Facilities

The above analysis was conducted assuming the case where all facilities are newly located, and it was confirmed that the alternative solutions appear around the optimal location. In the following, we will examine the problem assuming that facilities providing the same services already exist and that additional facilities are to be added to the existing facilities. We assume that there are existing facilities in 10 randomly selected municipalities. Figure 9(1) shows the distribution of the 10 randomly selected municipalities with existing facilities, and assuming that two additional facilities providing the same type of service are to be located under the given conditions, the optimal solution and the alternative solution (α % set based on the *R*-set) are obtained. The values of the objective function before and under the optimal solution are shown in Table 2. These values are used to set tolerance values when obtaining the alternative solutions.

Table 2. Objective function value of optimal solution (with existing facilities).

Number of facilities	Ave. dist. (km) *p*-median	Max. dist. (km) *p*-center	Coverage (people) MCLP
Existing facilities	14.399	52.066	23,860,970
Existing + New facilities	12.220	35.821	28,824,942

The results for the optimal solution and the alternative solutions are shown in Fig. 9. It can be seen that for all facility location problems, the alternative solutions are formed in the blank areas where no existing-facilities exist. The distribution of the nearest alternative solutions is skewed compared to the results obtained when there are no existing facilities. In particular, for the *p*-median and maximal covering location problems, the subset R_1 is spatially separated into two parts. The other facility can be selected from a very wide subset R_1. This means that there is a high degree of flexibility in the selection of candidate locations.

In the case where there are no existing facilities and all the facilities are to be newly located, the alternative solutions are distributed around the optimal solution, but when existing facilities are taken into account, they are formed even far from the optimal location. In other words, this indicates that when additional facilities are to be added to the existing facilities, it is necessary to study the facility location problem by referring to the spatial distribution of the candidate locations that can be used as an alternative solution, expanding the view to the entire target area, rather than sticking to the location of the optimal solution obtained in the conventional facility location problem.

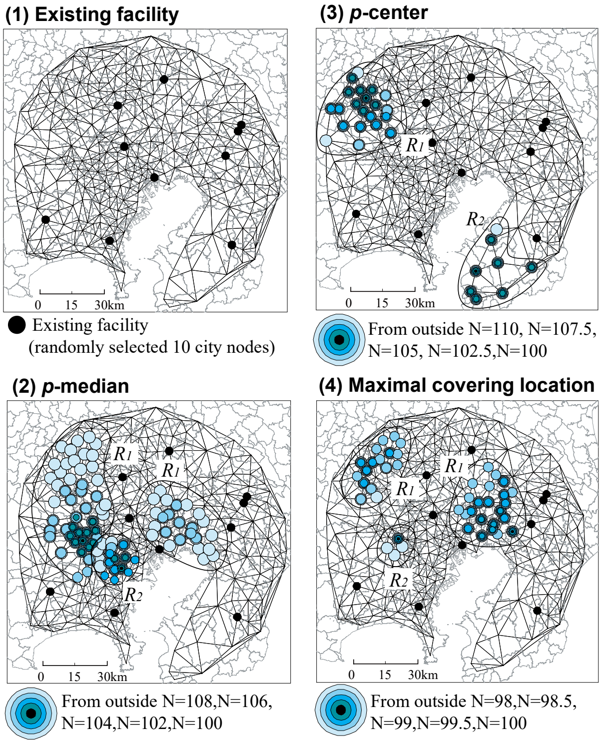

Figure 9. Alternative solutions based on the *R*-set (with existing facilities).

5 Extension of Alternative Solutions for Facility Location Planning

As an application of considering the alternative solutions of the facility location problem as a set of multiple candidate locations, we will consider location planning that simultaneously takes into account multiple determinants. Specifically, we consider a composite problem of the *p*-median problem and the maximal covering location problem. Figure 10(1) shows the results obtained by the *R*-set to obtain the 110% set for the *p*-median problem and the 90% set for the maximal covering location problem, and then integrating them by taking the two solutions. This integrated set is equivalent to the 108% set for the *p*-median problem and the 98% set for the maximal covering location problem as a result. Figure 10(2) shows the total cost that can be obtained by this integrated set. The darker shaded area in Fig. 10(2) can be considered to contain the Pareto optimum, which considers two different functions of the *p*-median problem and the maximal covering location problem. Pareto optimum refers to the state in which, when there are multiple evaluation indices, one evaluation index must be sacrificed to increase another. The *p*-median problem and the maximal covering location problem have different objective functions, so both cannot be evaluated by considering them simultaneously. However, by superimposing the alternative solutions of both problems, it is possible to support facility location planning while simultaneously considering multiple quantitative evaluation indices and even the problem of uncertainty, which is the main focus of this study.

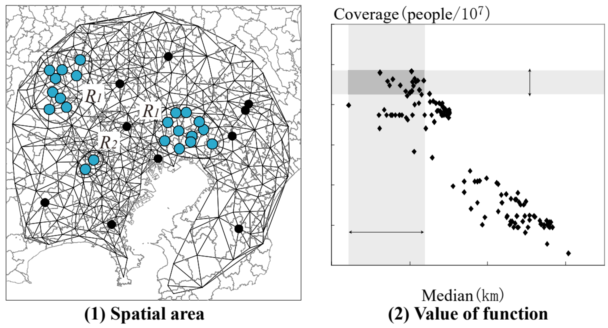

Figure 10. Alternative solutions based on the *R*-set and maximal covering location problems.

6 Summary and Conclusions

In this study, we first discussed the issues arising from uncertainty when applying the calculated optimal solution to actual facility location planning. To overcome this problem, we proposed a method that can increase the robustness of the facility location planning by finding alternative solutions (facility locations obtained by setting an acceptable range for the value of the objective function), thereby allowing more flexibility in candidate facility locations. Next, we conducted numerical analysis using hypothetical data on the spatial distribution of demands, and obtained knowledge on the characteristics of typical facility location problems. Furthermore, the characteristics of the optimal solution and the alternative solutions were examined using numerical analysis examples in which the proposed method was applied to real space, and the potential of the proposed method to find the alternative solutions was discussed. The main findings of this study are as follows.

As a result of obtaining the optimal solution and the alternative solutions in a simple hypothetical space and a real space, it was confirmed that the alternative solutions are formed in the vicinity of the optimal solution according to the R-set (α% set). In addition, a comparison of the alternative solutions of multiple facility location problems revealed that the spatial-inclusion relationship is relatively easy to establish for the alternative solutions of the p-median problem, and that a combination of arbitrarily selected locations among the candidate locations results in a relatively good evaluation value (total cost). However, it was found that the spatial-inclusion relationship is difficult to be established for the alternative solutions of the p-center problem and the maximal covering location problem.

We applied the proposed method to a real space and compared the alternative solutions of the p-median, p-center, and maximal covering location problems. As a result, it was found that the alternative solutions of the p-median problem tended to be concentrically distributed around the optimal location. The p-center problem showed a clear difference in the size of the alternative solutions (the number of candidate locations). It was found that the optimal locations of the maximal covering location problem tend to be similar to the p-median problem, however have the spatial distribution property that the alternative solutions extend in a specific direction.

When new facilities are to be added to the existing facilities, it was found that differences in the size of the alternative solutions (the number of candidate locations) tend to appear, and the alternative solutions may be formed in a region far from the optimal solution and spatially divided into multiple regions.

In the location planning of urban facilities, it is necessary to make decisions based on the various values held by many stakeholders and based on a wide variety of evaluation indicators. In such a process, it is difficult to discuss based on the only calculated optimal solution, and in some cases, it may become an obstacle to smooth discussion. The method for obtaining the alternative solutions proposed in this study (" α% sets" based on the R-set) can extract multiple sets of possible candidate locations, and can be effectively used in the actual facility location planning process.

Previous studies have primarily focused on quantitatively evaluating the impact of uncertainty on the optimal solution. In contrast, this study assumes that "uncertainty cannot be quantified in principle" and proposes a method for generating alternative solutions when the optimal solution cannot be implemented. We demonstrated the effectiveness of this approach through numerical examples. Planning processes of facility locations often require flexibility, redundancy, and robustness, as deviations from the original plan are inevitable. The method developed in this study directly addresses these fundamental requirements, making a significant contribution to the field.

However, this study has certain limitations and challenges. First, we considered a scenario in which demand locations are fixed to residential areas while uncertainty exists at the supply locations. However, since people continuously move within urban spaces using high-speed transportation networks, the location of demand locations also carries uncertainty. Therefore, future research should develop methods that account for uncertainty in demand locations as well.

Second, spatial movement costs in this study were represented using a simple distance-based metric. However, in reality, these costs involve multiple factors, including time, financial expenses, and physical effort. Moreover, uncertainties exist in traffic congestion and public transportation frequency, which should be considered in future studies.

Additionally, the redundancy of alternative solutions in this study was expressed using an index, "α%." Since the appropriate value of α likely depends on the type of facility being planned and the characteristics of the study area, further discussion is needed on how to determine its value effectively.

Data and Software Availability

Research data and code supporting this publication is available in the GitHub repository https:// https://github.com/Agile-DASA/2025/tree/main. Please follow the instructions in the “Notes on analysis process.pdf” in the repository. In this study, NUOPT provided by NTT DATA Mathematical Systems Inc. was used for solving for the K-set and R-set problems.

Declaration of Generative AI in writing

The authors declare that they have not used Generative AI tools in the preparation of this manuscript. Specifically, the AI tools were utilized for language editing but not for generating scientific content, research data, or substantive conclusions. All intellectual and creative work, including the analysis and interpretation of data, is original and has been conducted by the authors without AI assistance.

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