AGILE: GIScience Series, 6, 32, 2025. https://doi.org/10.5194/agile-giss-6-32-2025
Proceedings of the 28th AGILE Conference on Geographic Information Science, 10–13 June 2025.
Eds.: Auriol Degbelo, Serena Coetzee, Carsten Keßler, Monika Sester, Sabine Timpf, Lars Bernard
This contribution underwent peer review based on a full paper submission.

IGEO7: A new hierarchically indexed hexagonal equal-area discrete global grid system

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Abstract. Hexagonal Discrete Global Grid Systems (DGGS) offer significant advantages for spatial analysis due to their uniform cell shapes and efficient indexing. Among the three central place apertures (3, 4, and 7), aperture 7 subdivisions exhibit very desirable properties, including the preservation of hexagonal symmetry and the formation of unambiguous indexing hierarchies. Interest in hierarchically indexed aperture 7 hexagonal DGGS has recently increased due to the popularity of the H3 DGGS. But there are currently no open-source equal-area aperture 7 hexagonal DGGS available, that provide similar indexing capabilities like H3. We present IGEO7, a novel pure aperture 7 hexagonal DGGS, and Z7, its associated hierarchical integer indexing system. In contrast to H3, where cell sizes vary by up to $\pm 50\%$ across the globe, IGEO7 uses cells of equal area, making it a true equal-area DGGS. IGEO7 and Z7 are implemented in the open-source software DGGRID. We also present a use case for on-demand suitability modeling to demonstrate a practical application of this new DGGS.

Submission Type. theory; algorithm; software;

BoK Concepts. [DM3-2b] Grid representations; [DM3-4] The hexagonal model; [DM3-7] Hierarchical data models;

Keywords. igeo7, z7, dggs, spatial index

1 Introduction

Discrete Global Grid Systems (DGGS) have long been recognized as a meaningful alternative for organizing and analyzing global geospatial data (Goodchild and Shiren, 1992). DGGS are spatial reference systems that use a hierarchical tessellation of cells to partition and address the globe, unlike traditional 2-D cartographic projections, and have recently received increasing attention due to the popularity and wide adoption of the H3 DGGS (Brodsky, 2018). Because hexagonal grids have many advantages over other topologies, several hierarchically indexed hexagonal DGGS have been implemented (Sahr, 2011). In the literature, primarily DGGS are described using apertures 3 (Sahr, 2008) and 4 (Tong et al., 2010). But among the three central place apertures (3, 4, and 7), recursive aperture 7 groupings of hexagons preserve more of the inherent symmetry of hexagonal cells and best approximate a hexagonal shape across all resolutions (Sahr et al., 2003). Aperture 7 subdivision also forms unambiguous indexing hierarchies. H3 is an aperture 7 DGGS, but there is a growing need for equal-area alternatives that provide powerful hierarchical indexing and recursive subdivision capabilities similar to H3 (Kmoch et al., 2022b).

To effectively use such DGGS, adequate software tools must be available that can efficiently manipulate the unique geometry and indexing of these systems (Kmoch et al., 2022a). This article introduces the equal-area IGEO7 aperture 7 hexagonal DGGS and describes the hierarchical indexing Z7 that it uses.

2 Background and Related Work

2.1 DGGS Fundamentals

DGGS partition the Earth's surface into cells of approximately equal area to enable consistent spatial data indexing. A Platonic solid is used as the base polyhedron to approximate the Earth as a sphere. Platonic solids are unique polyhedra where all faces are identical regular polygons meeting at equal angles. The icosahedron is a common choice (Fig. 1). The construction of the DGGS is then guided by several steps (Sahr et al., 2003) that define the topology (cell shapes, neighbourhoods and hierarchical relationship), refinement level (aperture) and how the geometry of the DGGS is transformed from the planar faces of the base polyhedron to the sphere.

The most commonly used partition topologies are squares, triangles, diamonds, and hexagons. Many DGGS are based on polyhedra with triangular faces and are based on triangle-shaped cells. However, triangular cells have non-uniform neighbourhoods. Square and diamond shapes provide good compatibility with existing algorithms, but they also have non-uniform neighbourhoods. Hexagonal cells have the unique advantage of six neighbors with uniform adjacency.

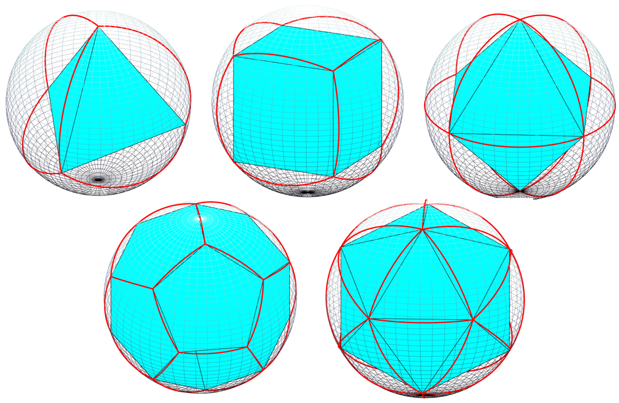

Figure 1. Spherical tessellations of five platonic solids. The icosahedron is in the lower right. (Lei et al., 2020, Figure 3, CC-BY-4.0)

A recursive partitioning method then subdivides the cells into smaller cells of the next finer resolution level; the number of cells in the finer resolution per cell in the coarser resolution is called the aperture or refinement ratio. Finally, a projection defines the mapping between the planar faces and the sphere. Kmoch et al. (2022b) have tested the area-preserving properties of several available open-source DGGS implementations and shown that H3, which uses a polyhedral gnomonic projection, introduces large areal distortions. The area of H3 cells varies by up to $\pm 50\%$ across the globe, whereas DGGS implementing the Icosahedral Snyder Equal Area (ISEA) or (r)HEALPIX projections ensured equal-area sized cells (Snyder, 1992; Górski et al., 2005). Thus, H3 is not suitable for use cases that require equal area cells.

2.2 Existing Indexing Approaches

Several DGGS indexing systems have been described in the literature or have been implemented in ready-to-use software packages. Some examples include the Quaternary Triangular Mesh (QTM), which pioneered hierarchically indexed Earth partitioning using a triangular tessellation and aperture 4 quadtree indexing (Dutton, 1996). HEALPix is an equal-area pixelization widely adopted in astronomy and climate science that implements a unique pixel identifier scheme, which also encodes its resolution (Górski et al., 2005). rHEALPix is built on a rearranged HEALPIX grid and extends its capabilities to ellipsoids of revolution and with hierarchical subdivision properties (Gibb et al., 2016). The ISEA3H equal area aperture 3 hexagonal grid (Sahr et al., 2003) has been implemented in several software packages, including the open source software package DGGRID (Sahr, 2024).

Eventually, the Open Geospatial Consortium's DGGS working group formalized a series of criteria and definitions on DGGS standards and interoperability in the OGC Abstract Specification Topic 21 (Open Geospatial Consortium, 2017).

There are several examples of hierarchical indexing systems for hexagonal DGGS. The ISEA3H family of indexing systems, developed by Sahr and extended by others, employs pure aperture-3 hexagonal grids based on the ISEA projection. The HQBS and related aperture 4 systems have also seen substantial algorithm development (Tong et al., 2010). Another example of hexagonal DGGS indexing systems described in the literature is the Lattice Quad-Tree Indexing (Zhou et al., 2020). In 2019, Uber open-sourced its hierarchically indexed hexagonal DGGS H3, with primarily aperture 7 indexes stored as 64-bit integers, gaining significant adoption in industry applications (Brodsky, 2018; Sahr, 2019). Each system presents distinct trade-offs between computational efficiency, spatial fidelity, and ease of implementation (Mahdavi-Amiri et al., 2015).

3 Methods and Implementation

IGEO7 is based on the icosahedron (Fig. 2). For IGEO7, the icosahedron uses the standard ISEA orientation that aims to minimize the number of icosahedron vertices on land while maintaining symmetry across the equator (Sahr et al., 2003, Figure 3c, p. 124). Like all icosahedral hexagonal grids, IGEO7 cells are primarily hexagons, but at each resolution, there are exactly twelve pentagonal cells centred on the vertices of the icosahedron. Because IGEO7 is using the ISEA projection, it shares the same beneficient equal-area properties like ISEA7H (Kmoch et al., 2022b, see Figures 1a, 5a, and 11a for equal-area comparison with H3).

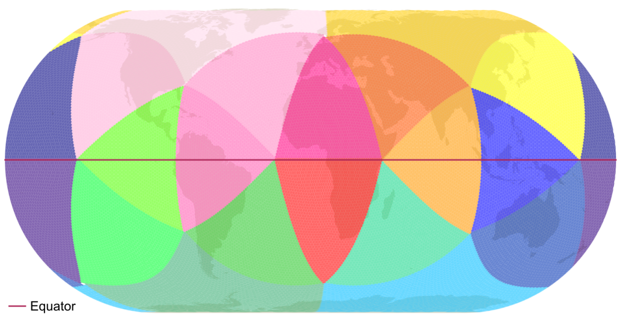

Figure 2. For IGEO7, the 20 triangular areas are the faces of the icosahedron. The locations where the vertices of the triangles meet are the 12 vertices of the icosahedron and indicate its orientation in relation to the Earth's surface.

Hierarchical integer indexes assign unique addresses across all resolutions, help preserve spatial locality in memory, and support hierarchical algorithms. IGEO7 is a pure aperture 7 icosahedral grid, and, similar to the indexing used in H3, the cells in IGEO7 are assigned hierarchical integer indexes using an aperture 7 Central Place Indexing Sahr (2019) approach called Z7.

The base cells of the Z7 indexing hierarchies correspond to the resolution 0 cells of an icosahedral DGGS, which are the 12 pentagons (Fig. 3) of the dual spherical dodecahedron. This differs from H3, where the resolution 0 cells are formed by initial aperture 3 and 4 refinements, which are not hierarchically indexed. This difference gives Z7 5 additional indexed resolutions.

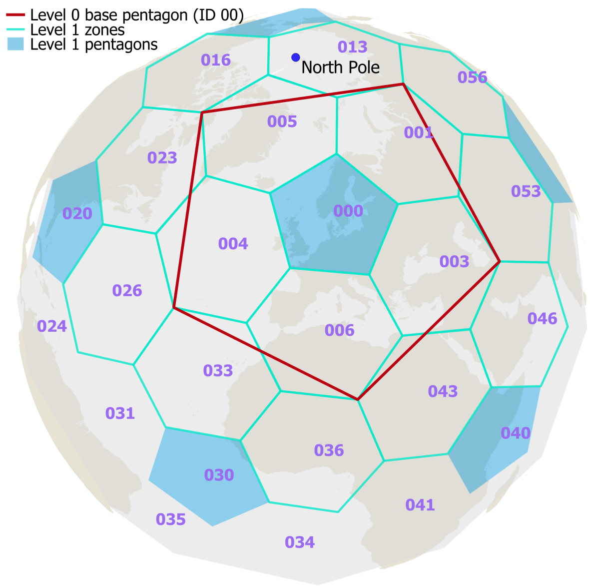

Figure 3. Initial configuration of IGEO7 - the 12 base pentagons with their centroids located at the vertices of the icosahedron are then recursively subdivided into hexagons.

A Z7 index is a 64-bit unsigned integer. The first four bits of the integer indicate the base cell number (0 to 11), and the remaining 60 bits encode the digits for each resolution, using 3 bits per resolution. Each digit has a value from 0 to 6, with a value of 7 used for digits greater than the resolution of the cell being indexed. The maximum Z7 resolution is resolution 20 (cf. Table 1). This allows for the highest resolution levels to encode precision up to ≈20cm (res. 18), ≈7.6cm (res. 19), and ≈2.9cm (res. 20).

Z7 indexing implements two external representations for input and output. The first is a hexadecimal integer, mimicking the external representation of H3 indexes. Z7 also provides a more human-legible string representation. This consists of two decimal digits for the quad number (00, 01, . . . , 11), followed by 20 octal digits (0 - 7), one per resolution of the cell being indexed (Fig. 4).

4 Illustrative Use Cases

Suitability modelling for the construction of hydrogen fuel stations

The HyTruck project helps public authorities in the Baltic Sea region design a network of hydrogen refuelling stations for large trucks, bringing the region closer to zero

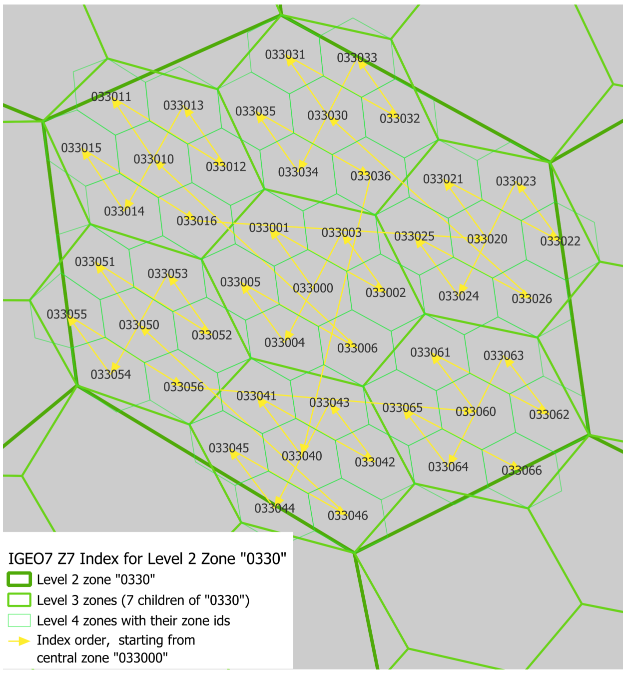

Figure 4. The index order defines a regular space-filling curve.

emissions in road freight transport (Bockler, 2023). To enable stakeholders to explore possible locations for constructing hydrogen refuelling stations, we implemented a dynamic suitability analysis decision support system. We collated various datasets, such as land use, slope, distances to main roads, distances to natural gas pipelines, and more, to model the required suitability criteria. The source datasets were prepared at a spatial resolution of 1 km through rasterisation. Subsequently, we quantized the source variables using IGEO7 at resolution 9 into hexagonal cells of approximately 1.3km diameter. We selected this resolution alignment to ensure minimal information loss during discretization. We organized the resulting data structure in tabular form, indexing it by their Z7 cell identifiers, with suitability variables stored as columns in the database.

The application processes spatial queries for the suitability assessment through dynamic translation from geographic coordinates to Z7 indices using the DGGRID software (Sahr, 2024). The Z7 indices are then used in database lookups to retrieve the variables for each grid cell.

Then, suitability criteria weights were derived through expert interviews and the analytical hierarchy process (Amrani et al., 2024). The weights for the variables are also stored in the database. When a request is issued for an area, the weights are applied during the data query for each cell, enabling efficient but, more so, very flexible computation of the suitability metrics. Our implementation facilitates multi-criteria analysis through customizable weighting schemes, which allows stakeholders to adjust priority parameters according to their planning objectives (Fig.5).

Table 1. The table shows several metrics about the number and approximate sizes of the cells at various refinement levels. The values are rounded to make the table more readable. The Characteristic Length Scale (CLS) is the diameter of a circle of the same area as a hexagon of the specified resolution. This is provided as a value more relatable with traditional raster resolutions. The CLS for levels 18-20 are ≈20cm (res. 18), ≈7.6cm (res. 19), and ≈2.9cm (res. 20).

Level	Cells	Area (km2)	CLS (km)	Hexagons	Pentagons	Area (m2)	CLS (m)
0	12	51006562.172	8199.500	0	12	51006562172408.9	8058757.5
1	72	7286651.739	3053.223	60	12	7286651738915.6	3045924.1
2	492	1040950.248	1151.643	480	12	1040950248416.5	1151251.1
3	3432	148707.178	435.153	3420	12	148707178345.2	435132.1
4	24012	21243.883	164.466	24000	12	21243882620.7	164464.4
5	168072	3034.840	62.162	168060	12	3034840374.4	62161.7
6	1176492	433.549	23.495	1176480	12	433548624.9	23494.9
7	8235432	61.936	8.880	8235420	12	61935517.8	8880.2
8	57648012	8.848	3.356	57648000	12	8847931.1	3356.4
9	403536072	1.264	1.269	403536060	12	1263990.2	1268.6
10	2824752492	0.181	0.479	2824752480	12	180570.0	479.5
11	19773267432	0.026	0.181	19773267420	12	25795.7	181.2
12	138412872012	0.004	0.068	138412872000	12	3685.1	68.5
13	968890104072	0.000	0.026	968890104060	12	526.4	25.9
14	6782230728492	0.000	0.009	6782230728480	12	75.2	9.8
15	47475615099432	0.000	0.004	47475615099420	12	10.7	3.7
16	332329305696012	0.000	0.001	332329305696000	12	1.5	1.4
17	2326305139872072	0.000	0.001	2326305139872060	12	0.2	0.5
18	16284135979104492	0.000	0.000	16284135979104480	12	0.00	0.00
19	113988951853731424	0.000	0.000	113988951853731412	12	0.000	0.000
20	797922662976120064	0.000	0.000	797922662976120052	12	0.000	0.000

Furthermore, we leverage the hierarchical structure of IGEO7's multi-resolution framework for systematic data aggregation. In the case study, resolution 9 is the highest resolution for which data is quantized. If a request queries data and suitability scores for a lower resolution, e.g. 7, it is easily possible to calculate all related child cell IDs up to resolution 9 based on the Z7 hierarchical indexing scheme. Through grouping and averaging the data variables and the suitability are finally aggregated into the resolution 7 cell. The application framework is written in Python and builds upon the open-source `dggrid4py` library, which provides a scripted high-level access to DGGRID's command-line functionalities (Kmoch and Chan, 2025).

This methodology allows stakeholders in different roles to reflect their priorities in planning and analysis with different weight settings. The implementation enables a customizable dynamic weighting and near-realtime on-demand assessment of each grid cell's suitability, while the underlying IGEO7 equal-area system guarantees that suitability scores are geographically consistent and directly comparable across large regions like Northern Europe, avoiding the significant analytical biases inherent in non-equal-area systems like H3.This use case demonstrates the flexible and much more rapid use of a DGGS in comparison to the classic raster-based map algebra in suitability mapping. Aligning the analysis with specific planning and decision-making objectives allows for a faster stakeholder feedback loop.

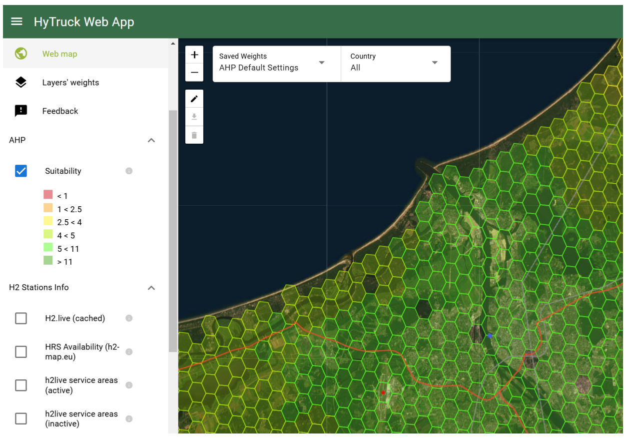

Figure 5. The hexagons show the weighted and aggregated suitability score over the Riga area in Latvia.

5 Discussion and conclusions

In this article, we introduced IGEO7, an aperture 7 hexagonal DGGS with its simple yet effective hierarchical integer indexing system, Z7. IGEO7, built upon the equal-area ISEA7H projection and the Z7 effective indexing system, can be considered as the first equal-area alternative to H3 - sharing the unique cell indices that carry resolution, size, and location information, but avoiding the areal distortions of H3. In the illustrated suitability modelling use case, it is important to rely on the assumption that cells represents the same ground area, providing an unbiased basis for comparing models and statistics – a consistency lacking in systems like H3 where substantial area distortions ($\pm 50\%$) can skew multi-criteria evaluations and aggregations.

Li and Stefanakis (2020) discuss the advantages of hexagonal DGGS over traditional geospatial indexing for data integration. Data from different spatial resolutions and reference systems are assigned to cells which share the same region. This process is called quantization and often relies on the distance between a cell centroid and a cell's immediate neighbour. Hexagonal hierarchical indexing offers representative sampling properties, especially if we consider spatial data as a continuous field sampled at cell centroids rather than as discrete bounded units. Li et al. (2021) show that in topographic analysis, hexagonal grids provide a more spatially uniform representation of the quantized data. Thus, hexagons represent spatial uncertainty and isotropy more reliably, which may outweigh the theoretical perfect containment properties offered by square, diamond or triangular systems, particularly for applications involving natural phenomena or distance-based analyses. Hexagons approach circular geometry more closely than any other tesselation shape and have uniform adjacency relationships. For both, IGEO7 and H3, where 7 child cells cover a parent cell, perfect spatial containment between resolutions is not possible with hexagonal tessellations – which may make them unsuitable for certain use cases – but this limitation may be less significant than traditionally emphasized.

Lastly, most DGGS are implemented on the sphere. Future work is planned to extend DGGRID and IGEO7 to ellipsoidal Earth models. However, research is needed on how this will affect the design and algorithmic use of IGEO7. We also envision more experimental work on different quantization methods, since currently, most available geospatial data remains in traditional raster and vector formats until remotely sensed Earth observation data is collected directly into a DGGS.

Declaration of Generative AI in writing

The authors declare that they have used Generative AI tools in the preparation of this manuscript. However, the AI tools were not utilized for generating scientific content, research data, or substantive conclusions. Specifically, AI tools were utilized in providing English language assistence for several sentences in the introduction and conclusion sections to provide more clarity. Beyond that, all intellectual and creative work, including the analysis and interpretation of data, is original and has been conducted by the authors without AI assistance.

Acknowledgements

This work was funded by the Estonian Research Agency (grant number PRG1764, PSG841), Estonian Ministry of Education and Research (Centre of Excellence for Sustainable Land Use (TK232)), and by the European Union (ERC, WaterSmartLand, 101125476, Interreg-BSR, HyTruck, #C031). Views and opinions expressed are, however, those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them. The authors are thankful for the technical support from the High Performance Computing Center of the University of Tartu.

Code and data availability

- The DGGRID software (Sahr, 2024) Github repository: https://github.com/sahrk/DGGRID
- dggrid4py, a Python library to run highlevel functions of DGGRID (Kmoch and Chan, 2025) Github repository: https://github.com/allixender/dggrid4py

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